Find $f(t)$ and a such that $1+\int_a^x \ln(f(t))\,dt = \frac{2}{x^3}$.

SCORE: _____ / 15 PTS

$$\frac{d}{dx}\left(1+\int_a^x \ln f(t)\,dt\right) = \frac{d}{dx}\left(\frac{2}{x^3}\right)$$

$$\ln f(x) = -6x^{-4}$$ 5

$$f(x) = e^{-6x^{-4}}$$

$$f(t) = e^{-6t^{-4}}$$ 2

$$1+\int_a^x \ln e^{-6t^{-4}}\,dt = \frac{2}{x^3}$$

$$1+\int_a^x -6t^{-4}\,dt = \frac{2}{x^3}$$

$$1+2t^{-3}\Big|_a^x = \frac{2}{x^3}$$

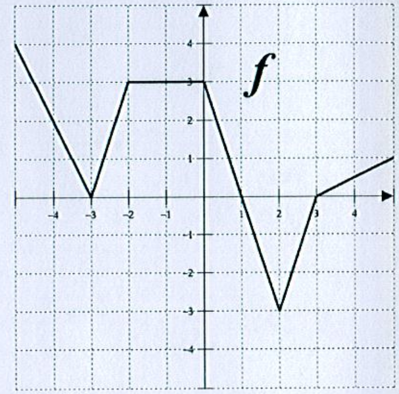
$$1+\cancel{2x^{-3}}-2a^{-3} = \cancel{\frac{2}{x^3}}$$ 5

$$\frac{1}{2} = a^{-3}$$

$$a = \sqrt[3]{2}$$ 3

Let $g(x) = \int_3^x f(t)\,dt$, where f is the function whose graph is shown on the right.

SCORE: _____ / 30 PTS



[a] Find $g(-4)$.

$\int_3^{-4} f(t)\,dt = -\int_{-4}^{3} f(t)\,dt$

$= - \left[\frac{1}{2}(2)(1) + \frac{1}{2}(2+4)(3) - \frac{1}{2}(3)(2)\right]$

$= -7$

② EACH

[b] Find $g'(-4)$. **<u>Explain your answer very briefly.</u>**

$g'(-4) = f(-4) = 2$ $2\frac{1}{2}$

[c] Find all critical numbers of g that do **<u>NOT</u>** correspond to local maxima nor local minima. **<u>Explain your answer very briefly.</u>**

$g'(x) = f(x) = 0$ BUT DOESN'T CHANGE SIGNS @ $x = -3$

$2\frac{1}{2}$ EACH

[d] Find all intervals over which g is increasing and concave down. **<u>Explain your answer very briefly.</u>**

$g'(x) = f(x) > 0$ AND DECREASING ON $(-5, -3)$ AND $(0, 1)$

$2\frac{1}{2}$ EACH

If $g(x)=\int_{x^2}^{2x} \ln(2+t^3)\,dt$, find $g''(1)$. **SCORE: _____ / 15 PTS**

$$g(x)=\int_{x^2}^{0}\ln(2+t^3)dt+\int_0^{2x}\ln(2+t^3)dt=\int_0^{2x}\ln(2+t^3)dt-\int_0^{x^2}\ln(2+t^3)dt$$

$$g'(x)=\ln(2+(2x)^3)\cdot 2-\ln(2+(x^2)^3)\cdot 2x=\underline{2}\,\underline{\ln(2+8x^3)}-\underline{2x}\,\underline{\ln(2+x^6)}$$

$$g''(x)=\underline{\frac{2\cdot 24x^2}{2+8x^3}}-\underbrace{2\ln(2+x^6)}_{1}-\underline{\frac{2x\cdot 6x^5}{2+x^6}}$$

$$g''(1)=\frac{48}{10}-2\ln 3-\frac{12}{3}$$

2 EACH EXCEPT AS INDICATED

$$=\underline{\frac{4}{5}-2\ln 3}$$

The table gives the acceleration of a car (in meters/minute^2) at various times (in minutes). **SCORE: _____ / 15 PTS**

At time $t = 2$, the velocity of the car was 7 meters/minute.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$a(t)$ ~~$v(t)$~~	3	8	6	5	2	4	7	9	10	13	11	12	15	14

[a] Write an expression involving an integral for the velocity of the car at $t = 10$.

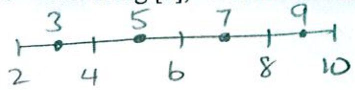
$$v(10) - v(2) = \int_2^{10} a(t)\,dt$$

$$v(10) = \underbrace{7}_{} + \underbrace{\int_2^{10} a(t)\,dt}_{} \quad (5)$$

(2)

[b] Estimate the velocity of the car at $t = 10$ using [a], 4 subintervals and midpoints. Specify the units of your answer.

$$\Delta t = \frac{10-2}{4} = 2$$



(4) (2)

$$7 + (a(3) + a(5) + a(7) + a(9))\Delta t = 7 + \overbrace{(5+4+9+13)}(2)\overbrace{}$$

$$(1) = \underbrace{69}_{}\ \underbrace{m/min}_{} \quad (1)$$

d

$$\int (3-4t)\sqrt{2t-5}\,dt$$

4

$$\underline{u = 2t-5}$$

$$du = 2\,dt$$

$$\tfrac{1}{2}\,du = dt$$

$$t = \frac{u+5}{2}$$

$$3-4t = 3-4\left(\frac{u+5}{2}\right)$$

$$= -2u-7$$

2

$$= \underbrace{\tfrac{1}{2}}\int \underbrace{(-2u-7)\sqrt{u}\,du}\ 5$$

$$= \int \left(-u^{\frac{3}{2}} - \tfrac{7}{2}u^{\frac{1}{2}}\right) du$$

$$= \underbrace{-\tfrac{2}{5}u^{\frac{5}{2}} - \tfrac{7}{2}\left(\tfrac{2}{3}\right)u^{\frac{3}{2}}}_{4} + C$$

$$= \underbrace{-\tfrac{2}{5}(2t-5)^{\frac{5}{2}} - \tfrac{7}{3}(2t-5)^{\frac{3}{2}}}_{3} \underbrace{+\,C}_{2}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \theta^2 \csc\theta \, d\theta$$

$\theta^2 \csc\theta$ IS DISCONTINUOUS

@ $\theta = 0$

3

SO FTC DOESN'T APPLY

2

$$\int \frac{14\sin 8y - 21e^{2y}}{\sqrt[3]{6e^{2y}+\cos 8y}}\,dy$$

4

$u = 6e^{2y} + \cos 8y$

$du = (12e^{2y} - 8\sin 8y)\,dy$

$= 4(3e^{2y} - 2\sin 8y)\,dy$

$-\frac{7}{4}du = -7(3e^{2y} - 2\sin 8y)\,dy$

$= (14\sin 8y - 21e^{2y})\,dy$

4 3

$$= \underbrace{-\frac{7}{4}}_{}\underbrace{\int \frac{1}{\sqrt[3]{u}}\,du}_{}$$

$$= -\frac{7}{4}\underbrace{\left(\frac{3}{2}\right)u^{\frac{2}{3}}}_{} + C$$

4

$$= \underbrace{-\frac{21}{8}(6e^{2y}+\cos 8y)^{\frac{2}{3}}}_{3} + \underbrace{C}_{2}$$

$$\int_{-2}^{2} \left(3\sqrt{16-(x-2)^2} - 2x^2 \sin x\right) dx$$

$$= 3\int_{-2}^{2}\sqrt{16-(x-2)^2}\,dx - \int_{-2}^{2} 2x^2\sin x\,dx \quad 2$$

$y=\sqrt{16-(x-2)^2}$

$(x-2)^2+y^2=16$

CIRCLE RADIUS 4 2

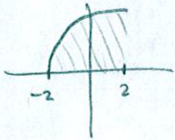
CENTER $(2,0)$

$2(-x)^2\sin(-x)$

$= -2x^2\sin x$ 2

2 ODD, CONTINUOUS 2

INTEGRAL $= 0$ 2



$$= 3\cdot\frac{1}{4}\pi(4)^2 \quad 3$$

$$= 12\pi \quad 2$$